TOPPER Sample Paper - I

Class : XI MATHEMATICS

Questions

Time Allowed : 3 Hrs Maximum Marks: 100

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1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section A

1. Evaluate $\lim\limits_{x\to 2} \frac{x-3}{x+4}$

2. Find the derivative of sin(x+1)

3. Find the truth value of p : "Every real number is either prime or composite."

4. Simplify $\frac{1+3i}{1-2i}$

5. A coin is tossed twice, find the probability of getting atleast one head.

6. Write the equation of the parabola whose focus is at (0, -4) and vertex is at (0, 0).

7. Identify the conic section represented by the equation $4x^2 + y^2 = 100$ and draw its rough graph.

8. Write the equation of a circle whose centre is at (2,-3) and radius is at 8.

9. Write the component statements of the given statement

P: Number 7 is prime or it is odd.

10. Identify the "OR" used in the statement:

An ice-cream or a coke free with a large pizza.

Section B

11. A and B are two sets such that $n(A-B) = 14+x$, $n(B-A) = 3x$ and $n(A \cap B) = x$, draw a Venn diagram to illustrate the information. If $n(A) = n(B)$, then find the value of x .

12. If the power sets of two sets are equal then show that the sets are also equal.

13. If f and g are two functions : $R \to R; f(x) = 2x - 1, g(x) = 2x + 3$, Then evaluate

(i) $(f + g)(x)$ (ii) $(f - g)(x)$ (iii) $(fg)(x)$ (iv) $\left(\frac{f}{g}\right)(x)$

14.Let R be a relation from N to N defined by R = { (a, b) ∈N and $a = b^4$ } Is the relation

(i) Reflexive (ii) Symmetric (iii) Transitive (iv) Equivalence

15. Express $(\cos\alpha + \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$ in terms cosine.

16. Comment on the nature of roots of the equation also find the roots
$27x^2 - 10x + 1 = 0$

17.Find the probability that when 7 cards are drawn from a well shuffled deck of 52 cards, all the aces are obtained.

18. From a group of 8 male teachers and 6 female teachers a committee of 8 is to be selected. In how many ways can this be done if it must consists of 3 men and 3 women?

19. In how many ways letters of the word "Mathematics " be arranged so that the (i) vowels are together (ii) vowels are not together

OR

In how many ways can 5 girls and 3 boys be seated in a row with 11 chairs so that no two boys are together?

20. A point M with x-coordinate 4 lies on the line segment joining the points P(2, –3, 4) and Q (8, 0, 10). Find the coordinates of the point M.

OR

Find the equation of the set of points such that the sum of the square of its distance from the points (3, 4, 5) and (-1, 3,-7) is a constant.

21. Find the general solution of the given trigonometric equation:
$\tan 2x + \sec^2 2x - 1 = 0$

OR

Solve for x: $\sin x + \sin 2x + \sin 3x = 0$

22 .Find the derivative of the given function :

$$\frac{4x + 5\sin x}{3x + 7\cos x}$$

OR

Find the derivative of the given function

$$y = \frac{x}{\sin^n x}$$

Section C

23. If $\frac{\pi}{2} \le x \le \pi$ and $\tan x = -\frac{4}{3}$, find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$.

24. Find the mean deviation about the median for the following data:

Marks	No. of students
0-10	5
10-20	10
20-30	20
30-40	5
40-50	10

25. Prove by the principle of Mathematical Induction that every even power of every odd integer greater than one when divided by 8 leaves remainder one.

26. Solve the following system of inequalities graphically:

$$x + 2y \le 10; x + y \ge 1; x - y \le 0; x \ge 0; y \ge 0$$

OR

For the purpose of an experiment an acid solution between 4% and 6% is required. 640 liters of 8% acid solution and a 2% acid solution is available in the laboratory. How many liters of the 2% solution need to be added to the 8% solution?

27.The first three terms in the binomial expansion of $(a+b)^n$ are given to be 729,7290 and 30375 respectively. Find a, b and n.

28.A student wants to buy a computer for Rs 12,000. He has saved up to Rs 6000 which he pays as cash. He is to pay the balance in annual installments of Rs 500 plus an interest of 12% on the unpaid amount. How much will the computer cost him?

OR

Find the value of $\dfrac{1\times 2^2+2\times 3^2+3\times 4^2+\ldots\text{uptill the nth term}}{1^2\times 2+2^2\times 3+3^2\times 4+\ldots\text{uptill the nth term}}$

29. Show that the equation of the line through the origin and making an angle of θ with the line y = mx + c is $\dfrac{y}{x}=\dfrac{m+\tan\theta}{1-m\tan\theta}$ or $\dfrac{y}{x}=\dfrac{m-\tan\theta}{1+m\tan\theta}$

Solutions to Sample Paper-1

Section A

1. $\lim_{x \to 2} \frac{x-3}{x+4} = \frac{2-3}{2+4} = \frac{-1}{6}$

[1 Mark]

2. [sin(x+1)]' = cos (x+1) .1 = cos (x+1)

[1 Mark]

3. Giving one counter example is enough to prove the false hood of a statement. Here counter example is: The real number 1 is neither prime nor composite. So the statement is false.

[1 Mark]

4. $\frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i} = \frac{1-6+3i+2i}{(1)^2-(2i)^2} = \frac{-5+5i}{1-4i^2} = \frac{-5+5i}{1+4} = \frac{-5+5i}{5} = -1+i$ [1 Mark]

5. S = {HH, HT, TH, TT} i.e. Total number of cases = 4

Favourable cases for atleast one head are { HH, HT, TH}.

Required probability $= \frac{3}{4}$

[1 Mark]

6. The equation of parabola whose focus is at (0,-a) and vertex (0,0) will is $x^2 = -4ay$

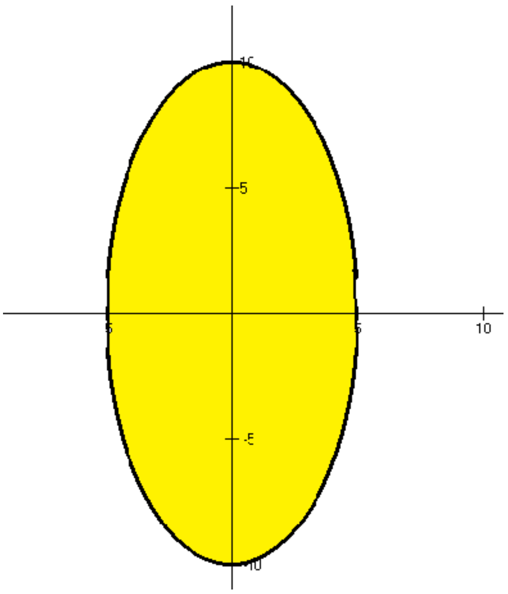
Here a=-4 so required equation is $x^2 = -16y$

[1 Mark]

7. $4x^2 + y^2 = 100$

$\frac{x^2}{25} + \frac{y^2}{100} = 1$

This is the equation of an ellipse with major axis along y axis



[1 Mark]

8. Equation of circle with centre (h,k) and radius r is

$(x-h)^2 + (y-k)^2 = r^2$

Here (h,k)=(2,-3) and r =8

So required equation is $(x-2)^2 + (y+3)^2 = 64$

[1 Mark]

9. The component statements are
p: 7 is a prime number. q: 7 is odd

[1 Mark]

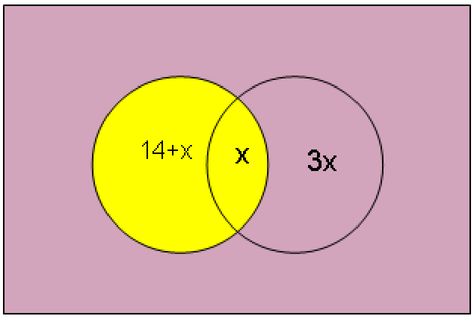
10. Exclusive use of the word “OR” since one can have either ice cream or coke but not both

[1 Mark]

Section B

11. n(A-B) = 14+ x , n(B-A) = 3x and n(A∩B) = x

[1 Mark]



n(A) = n(B)

n(A) = n(A-B) + n(A∩B) ; n(B) = n(B-A) + n(A∩B) [1Mark]

⇒ n(A-B) + n(A∩B) = n(B-A) + n(A∩B)

⇒ 14+x +x = 3x +x ⇒14 = 2 x ⇒x = 7 [2Marks]

12. Let a be any element that belongs to the set A , i.e a ∈ A

P(A) is the set of all subsets of the set A. Therefore {a} belongs to P(A)

i.e {a} ∈ P(A) [1Mark]

But P(A) = P(B) [Given]

∴ {a} ∈ P(B)

⇒ a ∈ B [1Mark]

So a ∈ A ⇒ a ∈ B so A ⊂ B

Similarly, $A \subset B$ [1Mark]

$\Rightarrow A = B$ [1Mark]

13. $f(x) = 2x - 1, g(x) = 2x + 3, ; x \in R$

$(f + g)(x) = f(x) + g(x) = (2x - 1) + (2x + 3) = 4x + 2; x \in R$ (1Mark)

$(f - g)(x) = f(x) - g(x) = (2x - 1) - (2x + 3) = -4$ (1Mark)

$(fg)(x) = f(x)g(x) = (2x - 1)(2x + 3) = 4x^2 - 2x + 6x - 3 = 4x^2 + 4x - 3$ (1Mark)

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x - 1}{2x + 3}; x \in R - \left\{-\frac{3}{2}\right\}$ (1Mark)

14. $\{(a,b), a = b^4, a, b \in N\}$

(i) $(a, a) \in R \Rightarrow a^4 = a$

which is true for a = 1 only, not for $a \in N$

$\therefore$ Relation is not reflexive [1mark]

(ii) $\{(a,b), a = b^4, a, b \in N\}$

and $\{(b,a), b = a^4, a, b \in N\}$

$a = b^4$ and $b = a^4$ cannot be true simultaneously

$\therefore$ Relation is not symmetric [1mark]

(iii) $\{(a,b), a = b^4, a, b \in N\}; \{(b,c), b = c^4, b, c \in N\}$

$\Rightarrow a = b^4 = c^{16}$

so $a \neq c^4$

$\therefore (a, c) \notin R$

$\therefore$ Relation is not Transitive [1mark]

Since the relation is not reflexive, not symmetric, not transitive

$\Rightarrow$ Relation is not an equivalence relation [1mark]

15. $(\cos\alpha + \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = \left[2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\right]^2 + \left[2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]^2$

[Using the formulae for (cos C + cos D) and (sin C - sin D)]

$= 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\alpha-\beta}{2}\right) + 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\sin^2\left(\frac{\alpha-\beta}{2}\right)$ [2marks]

$= 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\left[\cos^2\left(\frac{\alpha-\beta}{2}\right) + \sin^2\frac{\alpha-\beta}{2}\right]$ [1mark]

$= 4\cos^2\left(\frac{\alpha+\beta}{2}\right)$ (using $\sin^2\theta + \cos^2\theta = 1$) [1mark]

which is in terms of cosine of an angle .

16. $27x^2 - 10x + 1 = 0$

Comparing the given equation to a standard quadratic equation, $ax^2 + bx + c = 0$

$a = 27, b = -10, c = 1$ $\left(\frac{1}{2}\text{ mark}\right)$

$D = b^2 - 4ac = (-10)^2 - 4\times 27\times 1 = 100 - 108 = -8 < 0$ (1 mark)

Therefore the given quadratic equation has imaginary roots $\left(\frac{1}{2}\text{ mark}\right)$

roots are given by $x = \frac{-b\pm\sqrt{D}}{2a} = \frac{-(-10)\pm\sqrt{-8}}{2\times 27} = \frac{(10)\pm 2\sqrt{2}i}{54} = \frac{5\pm\sqrt{2}i}{27} = \frac{5}{27}\pm\frac{\sqrt{2}}{27}i$

So, the two roots are $\frac{5}{27}+\frac{\sqrt{2}}{27}i$ and $\frac{5}{27}-\frac{\sqrt{2}}{27}i$ (2 marks)

17. Total number of possible sets of 7 cards = ${}^{52}C_7$

Number of sets of 7 with all 4 aces = ${}^{4}C_4 \times {}^{48}C_3$ (1 mark)

(4 aces from among 4 aces and other 3 cards must be chosen from the rest 48 cards)

Hence Probability of 7 cards drawn containing 4 aces = $\frac{^{4}C_4 \times {}^{48}C_3}{^{52}C_7}$ (1 mark)

$= \frac{1}{7735}$ (1 mark)

18.The committee of 8 must contains atleast 3 men and 3 women

So committee can have 3 men 5 woman, 4 men 4 woman,5 men 3 woman

The total number of ways $= {}^{8}C_3 \times {}^{6}C_5 + {}^{8}C_4 \times {}^{6}C_4 + {}^{8}C_5 \times {}^{6}C_3$ [2marks]

$= 56 \times 6 + 70 \times 15 + 56 \times 20$

$= 336 + 1050 + 1120 = 2506$ [2 marks]

19. In “MATHEMATICS there are 11 letters of which 2 Ms, 2 As, 2 Ts

so total arrangements are $\frac{11!}{2!.2!.2!}$ = 4,989,600 (1 mark)

(i) In “MATHEMATICS” there are 4 vowels 2 As E and I

Since they must be together so “AAEI” is treated as a single unit

So there are 8 objects of which 2 Ms, 2 As, 2 Ts

So required number of arrangements are $\frac{8!}{2!.2!.2!}$ =5040 (2 marks)

(ii) Number of arrangements with vowels never together =

Total arrangements –arrangements in which vowels are never together

4,989,600-5040 = 4984560 (1 mark)

OR

First the 5 girls are arranged in 5! Ways as shown

$-G_1 - G_2 - G_3 - G_4 - G_5 -$

Now there are 6 places in which the boys can be arranged.

This can be done in $^6P_3$ ways [1 Mark]

$\Rightarrow$Total ways = $5! \times {}^6P_3$ [2 Marks]

= 120 x 6 x 5 x 4 = 14 , 400 [1 Mark]

20 Let M divide PQ in the ratio k : 1.
The coordinates of the point M are given by

[1 Mark]

here the x coordinate = $\frac{8k+2}{k+1} = 4.$ [1 Mark]

$\Rightarrow 8k + 2 = 4k + 4 \Rightarrow 4k = 2 \Rightarrow k = \frac{2}{4} = \frac{1}{2}$ [1 Mark]

Putting k back in the x , y and z coordinate of the point M, we have (4,-2,6) [1 Mark]

OR

Let the given points be A and B : A(3,4,5) and B (-1,3 -7). Let the required point be P : P(x,y,z)

Given : $PA^2 + PB^2 = k^2$(a constant) [1 Mark]

$\Rightarrow (x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = k^2$ [1 Mark]

$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 + z^2 - 10z + 25 + x^2 + 2x + 1 + y^2 - 6y + 9 + z^2 + 14z + 49 = k^2$ [1 Mark]

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 - k^2 = 0$ [1 Mark]

This is the equation of the set of points P that satisfy the condition

21. $\tan 2x + \sec^2 2x - 1 = 0$

$\Rightarrow \tan 2x + 1 + \tan^2 2x - 1 = 0$

$\Rightarrow \tan 2x + \tan^2 2x = 0$

$\Rightarrow \tan 2x[1 + \tan 2x] = 0$ [1 Mark]

$\Rightarrow \tan 2x = 0$ or $1 + \tan 2x = 0$

$\Rightarrow \tan 2x = 0$ or $\tan 2x = -1$

$\tan 2x = 0 \Rightarrow 2x = n\pi \Rightarrow x = \frac{n\pi}{2}$ [1 Mark]

$\tan 2x = -1 \Rightarrow \tan 2x = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

$\Rightarrow 2x = n\pi + \frac{3\pi}{4}, n \in Z$

$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}, n \in Z$ [2 Marks]

OR

$\sin x + \sin 2x + \sin 3x = 0$

$\sin x + \sin 3x + \sin 2x = 0$

$2\sin\left(\frac{x + 3x}{2}\right)\cos\left(\frac{x - 3x}{2}\right) + \sin 2x = 0$

$2\sin 2x \cos(-x) + \sin 2x = 0$

$\sin 2x(2\cos x + 1) = 0$ (2 marks)

$\sin 2x = 0$ or $\cos x = \frac{-1}{2} = \cos(\pi - \frac{\pi}{3})$ (1 mark)

$\sin 2x = 0 \Rightarrow 2x = n\pi$

or $\cos x = \frac{-1}{2} \Rightarrow x = 2n\pi \pm (\pi - \frac{\pi}{3})$

$x = \frac{n\pi}{2}$ or $x = 2n\pi \pm (\pi - \frac{\pi}{3})$ (1 mark)

22. $$\frac{d}{dx}\left[\frac{4x+5\sin x}{3x+7\cos x}\right] = \frac{(3x+7\cos x)\frac{d}{dx}(4x+5\sin x)-(4x+5\sin x)\frac{d}{dx}(3x+7\cos x)}{(3x+7\cos x)^2}$$

$$= \frac{(3x+7\cos x).(4+5\cos x)-(4x+5\sin x).(3-7\sin x)}{(3x+7\cos x)^2}$$ [2 Marks]

$$= \frac{4(3x+7\cos x)+5\cos x(3x+7\cos x)-3(4x+5\sin x)+7\sin x(4x+5\sin x)}{(3x+7\cos x)^2}$$

$$= \frac{(12x+28\cos x)+(15x\cos x+35\cos^2 x)-(12x+15\sin x)+(28x\sin x+35\sin^2 x)}{(3x+7\cos x)^2}$$

$$= \frac{15(x\cos x-\sin x)+28(\cos x+x\sin x)+35(\sin^2 x+\cos^2 x)}{(3x+7\cos x)^2}$$

$$= \frac{15(x\cos x-\sin x)+28(\cos x+x\sin x)+35}{(3x+7\cos x)^2}$$ [2 Marks]

OR

$$y = \frac{x}{\sin^n x}$$

$$\frac{dy}{dx} = \frac{\sin^n x \frac{d}{dx}(x) - x\frac{d}{dx}(\sin^n x)}{\sin^{2n} x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^n x.1 - xn(\sin^{n-1} x).\cos x}{\sin^{2n} x}$$ [2 Marks]

$$\Rightarrow \frac{dy}{dx} = \frac{(\sin^{n-1} x)\left[\sin x - xn.\cos x\right]}{\sin^{2n} x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left[\sin x - xn.\cos x\right]}{\sin^{2n-n+1} x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x - nx\cos x}{\sin^{n+1} x}$$ [2 Marks]

<u>Section C</u>

23. $\tan x = -\frac{4}{3}$; $\frac{\pi}{2} \le x \le \pi$

We know that $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\Rightarrow \tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

$\Rightarrow -\frac{4}{3} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \Rightarrow 4\left(1-\tan^2\frac{x}{2}\right) = -6\tan\frac{x}{2}$ [1 Mark]

$\Rightarrow -4\tan^2\frac{x}{2} + 6\tan\frac{x}{2} + 4 = 0 \Rightarrow 4\tan^2\frac{x}{2} - 6\tan\frac{x}{2} - 4 = 0 \Rightarrow 2\tan^2\frac{x}{2} - 3\tan\frac{x}{2} - 2 = 0$

The equation is quadratic in $\tan\frac{x}{2}$

$\Rightarrow \tan\frac{x}{2} = \frac{-(-3) \pm \sqrt{9+16}}{2.2} = \frac{3 \pm 5}{4} = 2, -\frac{1}{2}$ [2 Mark]

Given $\frac{\pi}{2} \le x \le \pi \Rightarrow \frac{\pi}{4} \le \frac{x}{2} \le \frac{\pi}{2} \Rightarrow \frac{x}{2} \in$ I quadarnt

In I quadrant, $\tan\frac{x}{2} \ge 0 \Rightarrow \tan\frac{x}{2} = 2$ [1 Mark]

We know, $1+\tan^2\theta = \sec^2\theta$

$\Rightarrow 1+\tan^2\frac{x}{2} = \sec^2\frac{x}{2} \Rightarrow 1+(2)^2 = \sec^2\frac{x}{2} \Rightarrow \sec^2\frac{x}{2} = 5 \Rightarrow \sec\frac{x}{2} = \pm\sqrt{5} \Rightarrow \cos\frac{x}{2} = \pm\frac{1}{\sqrt{5}}$ [1 Mark]

We know $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\sin\frac{x}{2} = \pm\sqrt{1-\cos^2\frac{x}{2}} = \pm\sqrt{1-\frac{1}{5}} = \pm\sqrt{\frac{4}{5}} = \pm\frac{2}{\sqrt{5}}$ [1 Mark]

$\therefore$ (i) $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

(ii) $\cos\frac{x}{2} = \frac{1}{\sqrt{5}}$

(iii) $\tan\frac{x}{2} = 2$

24.

Marks	Frequency (f_i)	Cumulative frequency (cf)
0-10	5	5
10-20	10	15
20-30	20	35
30-40	5	40
40-50	10	50
Total	N= 50	

$$\text{Median (M)} = \ell + \frac{\frac{n}{2} - cf}{f} x h$$

ℓ = lower limit of median class, n = number of observations,

cf = cumulative frequency of class preceding median class, h = class size and f = frequency of median class

Substituting the values we get

$$\text{Median (M)} = 20 + \frac{(25-15)\times 10}{20}$$

Median (M) = 25 (2 marks)

xi	fi	$\lvert di \rvert = \lvert x_i - M \rvert$	$f_i \lvert d_i \rvert$
5	5	20	100
15	10	10	100
25	20	0	0
35	5	10	50
45	10	20	200
	50		450

.[2Marks]

$\therefore \sum_{i=1}^{n} |d_i| f_i = 450, \quad n = \sum_{i=1}^{n} f_i = 50$

$\therefore$ M.D (M) $= \dfrac{\sum_{i=1}^{n} |d_i| f}{n} = \dfrac{450}{50} = 9$ . ...[2 Marks]

25. The first odd integer > 1 , is 3 .
The general term for odd number >1 is (2 r +1)
P(n) : $(2r+1)^{2n} = 8m + 1$ where m, n are natural numbers
i.e P(n) : $(2r+1)^{2n} - 1$ is divisible by 8 [1 Mark]
Here P(1): $(2r+1)^{2.1} - 1$ is divisible by 8.
Consider $(2r+1)^2 - 1 = 4r^2 + 4r = 4r(r+1)$
r(r+1) being the product of consecutive natural numbers is even so
4r(r+1) is divisible by 8
Therefore, P(1) is true [1 Marks]

Let us assume P(k) to be true
P(k) : $(2r+1)^{2k} - 1$ is divisible by 8. [1 Mark]

Using this assumption, we will prove P(k+1) to be true
P(k+1) : $(2r+1)^{2(k+1)} - 1$ is divisible by 8.
Consider $(2r+1)^{2(k+1)} - 1 = (2r+1)^{2k} (2r+1)^2 - 1 = (8m+1)(8p+1) - 1$
[using P(1) and P(k), where m and p are integers]
$(2r+1)^{2(k+1)} - 1 = 64mp + 8(m+p) + 1 - 1$
$64mp + 8(m+p)$
Which is divisible by 8 [2 Marks]
Thus P(k+1) is true whenever P(k) is true , Also P(1) is true
$\Rightarrow$P(n) is true for every natural number n. [1 Mark]

26. $x + 2y = 10$ or $x = 10 - 2y$

x	14	10	6
y	-2	0	2

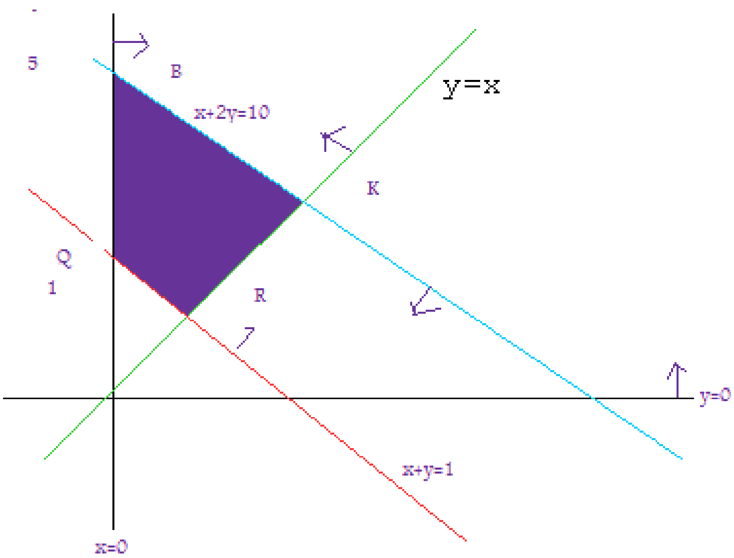
$x + y = 1$ or $y = 1 - x$

x	-2	0	3
y	3	1	-2

x – y = 0 or y = x

x	-2	0	2
y	-2	0	2

(2 Marks)



(4 marks)

26.The amount of acid in 640 litres of the 8% solution = 8% of 640 = $\frac{8 \times 640}{100}$

Let x litres of the 2% solution be added to obtain a solution between 4% and 6%

The amount of acid in 2% of x litres of acid = $\frac{2 \times x}{100}$

The resultant amount = 640 + x [1 Mark]

The amount of acid in (640 + x) litres solution is = $\frac{8 \times 640}{100} + \frac{2 \times x}{100}$

Acid percentage of the solution now = $\frac{\frac{8 \times 640}{100} + \frac{2 \times x}{100}}{640 + x} \times 100$ [1 Mark]

$\Rightarrow 4 < \frac{\frac{8 \times 640}{100} + \frac{2 \times x}{100}}{640 + x} \times 100 < 6$ [1 Mark]

$\Rightarrow \frac{4(640 + x)}{100} < \frac{8 \times 640}{100} + \frac{2 \times x}{100} < \frac{6(640 + x)}{100}$

$\Rightarrow 4(640 + x) < 5120 + 2x < 6(640 + x)$

$\Rightarrow 2(640 + x) < 2560 + x < 3(640 + x)$

$\Rightarrow 2(640 + x) < 2560 + x < 3(640 + x)$

$\Rightarrow 2(640 + x) < 2560 + x$ and $2560 + x < 3(640 + x)$

$\Rightarrow x < 1280$ and $320 < x$

$\Rightarrow 320 < x < 1280$ [3 Mark]

27.The first three terms in the binomial expansion $(a+b)^n$, ie t_1, t_2, t_3 are given.

$\Rightarrow t_1 = {}^nC_0 a^n b^0 = 729 \ldots (i);$

$t_2 = {}^nC_1 a^{n-1} b^1 = 7290 \ldots. (ii);$

$t_3 = {}^nC_2 a^{n-2} b^2 = 30375 \ldots. (iii)$ [1 Mark]

Now, $t_1 = {}^nC_0 a^n b^0 = 729 \Rightarrow 1 \times a^n \times 1 = 729 \Rightarrow a^n = 729 \ldots (iv)$

Dividing(ii)by(i), wehave

$$\frac{t_2}{t_1} = \frac{{}^nC_1 a^{n-1} b^1}{{}^nC_0 a^n b^0} = \frac{7290}{729} = 10 \Rightarrow \frac{na^{n-1}b}{a^n} = 10 \Rightarrow \frac{nb}{a} = 10 \ldots. (v)$$ [1 Mark]

Multiplying(iii)by(i), wehave

$$t_3 \times t_1 = {}^nC_2 a^{n-2} b^2 \times {}^nC_0 a^n b^0 = \frac{n(n-1)}{2} a^{2n-2} b^2 = 729 \times 30375 \ldots (vi)$$ [1 Mark]

Squaring (ii), wehave

$$\left[{}^nC_1 a^{n-1} b^1\right]^2 = [7290]^2 \Rightarrow n^2 a^{2n-2} b^2 = 7290 \times 7290 \ldots (vii)$$

Dividing(vi)by(vii), wehave

$$\frac{\frac{n(n-1)}{2} a^{2n-2} b^2}{n^2 a^{2n-2} b^2} = \frac{729 \times 30375}{7290 \times 7290}$$

$$\Rightarrow \frac{(n-1)}{2n} = \frac{30375}{7290 \times 10}$$

$$\Rightarrow \frac{(n-1)}{2n} = \frac{5}{12}$$

$\Rightarrow 12n - 12 = 10n \Rightarrow 2n = 12 \Rightarrow n = 6$

Putting n = 6 in (iv), we have

$a^6 = 729 \Rightarrow a = 3$

Putting n = 6, a = 3 in (v), we have

$$\frac{6b}{3} = 10 \Rightarrow b = 5$$

Hence, $a = 3, b = 5, n = 6$ [3 Marks]

28. Interest to be paid with Installment 1 (SI. On Rs 6000 for 1 year) =

$$\frac{6000 \times 12 \times 1}{100} = 720$$

Interest to be paid with Installment 2 (SI. On Rs 5500 for 1 year) =

$$\frac{5500 \times 12 \times 1}{100} = 660$$

Interest to be paid with Installment 3(SI. on Rs 5000 for 1 year) =

$$\frac{5000 \times 12 \times 1}{100} = 600$$

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Interest to be paid with Installment 1st (SI. On Rs 500 for 1 year) =

$$\frac{500 \times 12 \times 1}{100} = 60$$

Total interest paid = 720+660 + 600+....+ 60 (3 marks)

This forms an AP , with a1 = 720 and d = =-60 (1 mark)

$$Sn = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow Sn = \frac{12}{2}\left[2 \times 720 + (12-1)(-60)\right] = 4680$$

The computer costed the student = 12000+ 4680= 16680 (2 marks)

OR

$$\frac{1\times 2^2 + 2\times 3^2 + 3\times 4^2 + \ldots\text{.uptill the nth term}}{1^2\times 2 + 2^2\times 3 + 3^2\times 4 + \ldots\text{.uptill the nth term}}$$

onsider Numerator $= 1\times 2^2 + 2\times 3^2 + 3\times 4^2 + \ldots. + \text{uptill the nth term}$

he nth term is $n\,(n+1)^2$

$1\times 2^2 + 2\times 3^2 + 3\times 4^2 + \ldots\text{.uptill the nth term} = \sum n\,(n+1)^2$

onsider Denominator $= 1^2\times 2 + 2^2\times 3 + 3^2\times 4 + \ldots\text{.uptill the nth term}$

he nth term is $n^2\,(n+1)$

$1^2\times 2 + 2^2\times 3 + 3^2\times 4 + \ldots\text{.uptill the nth term} = \sum n^2\,(n+1)$

ow,Numerator $= \sum n\,(n+1)^2 = \sum n\,(n^2+1+2n) = \sum\,(n^3+n+2n^2) = \sum n^3 + 2\sum n^2 + \sum n$ [1 Mark]

$\Rightarrow$ Numerator $= \left[\frac{n(n+1)}{2}\right]^2 + 2\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + 2\frac{(2n+1)}{3} + 1\right]$

$\frac{n(n+1)}{12}\left[3n^2 + 3n + 8n + 4 + 6\right] = \frac{n(n+1)}{12}\left[3n^2 + 11n + 10\right] = \frac{n(n+1)}{12}\left[(3n+5)(n+2)\right]$.....[$1\frac{1}{2}$ Mark]

ow,Denominator $= \sum n^2\,(n+1) = \sum\,(n^3+n^2) = \sum n^3 + \sum n^2$ [1 Mark]

$\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{(2n+1)}{3}\right]$

$\frac{n(n+1)}{12}\left[3n^2 + 3n + 4n + 2\right] = \frac{n(n+1)}{12}\left[(3n+1)(n+2)\right]$ [$1\frac{1}{2}$ Mark]

he given expression $= \dfrac{\frac{n(n+1)}{12}\left[(3n+5)(n+2)\right]}{\frac{n(n+1)}{12}\left[(3n+1)(n+2)\right]} = \dfrac{3n+5}{3n+1}$ [1 Mark]

29. Of the given line y = mx + c, the slope is m

Let m_1 be the slope of the required line

$\tan\theta = \left|\frac{m_1 - m}{1 + m m_1}\right| \Rightarrow \tan\theta = \pm\frac{m_1 - m}{1 + m m_1}$ [1Mark]

Case I When , $\tan\theta = \frac{m_1 - m}{1 + m m_1}$

$\Rightarrow \tan\theta(1 + m m_1) = (m_1 - m) \Rightarrow \tan\theta + m m_1 \tan\theta = m_1 - m \Rightarrow m_1(m\tan\theta - 1) = -(m + \tan\theta)$

$\Rightarrow m_1(1 - m\tan\theta) = (m + \tan\theta) \Rightarrow m_1 = \frac{(m + \tan\theta)}{(1 - m\tan\theta)}$ [2Marks]

The equation of the line , through the origin is y-0 = $\left[\frac{(m + \tan\theta)}{(1 - m\tan\theta)}\right](x - 0)$

$\Rightarrow y = \left[\frac{(m + \tan\theta)}{(1 - m\tan\theta)}\right]x \Rightarrow \frac{y}{x} = \frac{(m + \tan\theta)}{(1 - m\tan\theta)}$ [1Mark]

Case I When , $\tan\theta = -\frac{m_1 - m}{1 + m m_1}$

$\Rightarrow \tan\theta(1 + m m_1) = (m - m_1) \Rightarrow \tan\theta + m m_1 \tan\theta = m - m_1 \Rightarrow m_1(m\tan\theta + 1) = (m - \tan\theta)$

$\Rightarrow m_1 = \frac{(m - \tan\theta)}{(1 + m\tan\theta)}$ [1Mark]

The equation of the line , through the origin is y-0 = $\left[\frac{(m - \tan\theta)}{(1 + m\tan\theta)}\right](x - 0)$

$\Rightarrow y = \left[\frac{(m - \tan\theta)}{(1 + m\tan\theta)}\right]x \Rightarrow \frac{y}{x} = \frac{m - \tan\theta}{1 + m\tan\theta}$ [1Mark]

Thus the equation of the line is $\frac{y}{x} = \frac{m + \tan\theta}{1 - m\tan\theta}$ or $\frac{y}{x} = \frac{m - \tan\theta}{1 + m\tan\theta}$